Sound clinical judgments derive both from the command of a sufficient body of facts and from the skill to combine such facts appropriately. Most undergraduate and graduate medical education concentrates on the first of these elements, the acquisition of knowledge; little formal effort is directed to the logic of dealing with clinical problems.

In this discussion we suggest that the theory and technics of decision analysis provide new and useful strategies appropriate for dealing with complex clinical situations. In their qualitative aspects these formal strategies closely resemble those that the expert clinician employs informally, but which he is often unable to communicate explicitly. When applied quantitatively, the formalism affords greater precision than is otherwise readily attainable.

To illustrate the application and utility of decision analysis we have considered the problems posed by severely hypertensive patients with possible functional renal artery stenosis, and have examined, both qualitatively and quantitatively, the alternative courses of action available to the clinician.

Although decision making is the preeminent function of the physician, medical education has paid remarkably little attention to the nature of the decision-making process. Medical school and postgraduate training emphasizes instead the acquisition of specific factual data and an understanding of pathophysiologic mechanisms. It also attempts to instruct the student in how a combination of specific findings constitutes the constellation of a disease and teaches him modes of therapy and the potential values and dangers of various forms of treatment. These considerations are essential, but the medical curriculum must be judged not merely by the technical skills imparted, nor by the interest of the facts in their own right, but also by the ability of the physician to combine diverse data to reach appropriate decisions concerning management. Yet nowhere is the student or graduate physician exposed to a systematic exposition of procedures for good decision making. Rather, it is generally believed that with experience the physician will somehow acquire the precious attribute of "clinical judgment."

The paramount importance of decision making in clinical medicine has prompted discussions of ways in which the process might be improved. But attention has been directed al-
most exclusively towards the logical structure of the diagnostic process [1–5]. Most investigators have virtually ignored the more complex problem of how risks and benefits should be integrated into decisions concerning the choice of both diagnostic procedures and modes of therapy. Yet the essence of clinical judgment resides in the ability of the physician to weigh the advantages and disadvantages of a diagnostic or therapeutic procedure and to choose a course of action for a particular patient based on estimates of such costs and benefits.

During recent years, practical strategies have been developed for balancing risks and benefits in making decisions under conditions of uncertainty. These have evolved into the discipline of decision analysis [6], which is now finding application in industrial management, economics and government. However, only a few studies [7–9] have attempted to apply decision analysis to medicine, and these have had little impact because attention to the specific clinical problem in question has obscured the general applicability of the procedure and because the complexities of the numerical calculations involved in the formal application of the technique have made the approach forbidding to clinicians.

Because we believe that the importance of decision analysis for clinical medicine has not been adequately appreciated, we consider here the qualitative aspects of decision analysis and attempt to demonstrate that, even in its nonmathematical form, decision analysis has important potential value to the physician. We also examine, using a common clinical problem, the quantitative approach to decision analysis and the ways in which this formalism may be useful in medical care.

BACKGROUND CONSIDERATIONS

Before considering the character of decision analysis, it is well to examine how a competent physician makes decisions concerning diagnosis or treatment. Typically, he first evaluates symptoms and signs and then turns for further help to the results of simple routine laboratory procedures. These narrow the range of diagnostic possibilities and may even permit him to establish a diagnosis. Often, however, the diagnosis is still unclear, and further tests seem indicated. But these further tests often carry greater risks and discomforts than the original screening procedures, and the physician must decide whether the information to be gained is sufficient to justify subjecting the patient to these more strenuous studies. Presuming now that he has established a diagnosis with a reasonable degree of certainty, the physician must select a treatment. Once again he will weigh several factors, some involving the likelihood of improvement or cure, others the possibility of harm. Again a variety of considerations, including value judgments, enter into his decision.

How well the physician carries out his exercise in diagnosis and management depends on his medical knowledge and how he applies it. Obviously, he must have sufficient information about the disease being considered, i.e., the risks of various diagnostic and therapeutic procedures and the likelihoods of various outcomes. Beyond this, however, he must also know how to assemble the facts so that he can logically compare alternative courses of action and assess their immediate and long-term consequences.

Let us consider for example the management of a patient with severe hypertension in whom diagnoses other than essential hypertension and functional renal artery stenosis have, for all practical purposes, been excluded. Many physicians faced with this problem order a renal arteriogram routinely. In the following section of this discussion, we shall investigate the appropriateness of this decision in some detail because it involves considerations of factors common to a large number of important clinical problems. Two options are available to the physician confronted with a patient in whom he suspects functional renal artery stenosis: (1) He can recommend an arteriogram and measurement of renal venous renin with the view that if the results are positive he will subject the patient to surgical repair of the renal artery. (2) He can proceed immediately to medical treatment without carrying out further studies. The full complexity of this decision becomes apparent from a consideration of such issues as the following: What is the risk of arteriography and of catheterization of the renal veins? How reliable will the data be? If the data suggest a functionally significant lesion, what are the risks associated with surgical repair? How likely is the blood pressure to be restored to normal by surgery as compared to drug therapy? What are the risks of drug therapy? What is the likelihood that, despite the control of blood pressure by drug therapy, the continued narrowing of the renal artery will lead to infarction of a kidney? What is the patient’s attitude towards surgery as compared to his attitude towards long-term treatment with antihypertensive medications?

Thus, the question of whether to obtain an arteriogram or to proceed immediately with drug therapy can be answered competently only by an orderly and careful consideration of a variety of important issues. Some physicians, whether by instinct or experience, are able to construct in their
minde an appropriate framework for dealing with such problems. Others, however, are less able to do so and might therefore benefit from formal exposure to a systematic approach to decision making.

PROBLEM REPRESENTATION: THE DECISION TREE
In deciding whether to employ either drugs or diagnostic studies and possible surgery, the physician realizes that the consequences of his actions are generally not under his precise control. For example, his diagnosis may be wrong and treatment may therefore fail. Alternatively, even if the diagnosis is correct and the treatment appropriate, the therapy may be ineffective or even injurious if complications occur. The physician must therefore recognize that he has only limited control over consequences and that his decisions as to present action may limit his future options. If he chooses one action over another, the second may be no longer viable, if later desired, because the first has failed. What is needed, therefore, is a systematic means of representing the relationship of present actions to long-term consequences. This step of organization and representation is the first step in analysis, helping the physician to ensure that he will not overlook any significant possibilities.

One particular form in which decision problems can be represented effectively is called a decision tree. We present such a tree for the hypertension problem discussed (Figure 1). It is obvious that the definition of actions and consequences is a matter of judgment. Sufficient detail must be included to make the diagram realistic, but the diagram must not be allowed to grow so large that it becomes unmanageable.

A decision tree consists of nodes and branches. There are two kinds of nodes. At decision nodes, symbolized in Figure 1 by circles, the physician must choose one from a set of actions. At chance nodes, indicated by squares, the outcome is not under the physician's control. For example, at decision node 1 in Figure 1 the physician must choose either studies or drugs. If he chooses studies rather than medical treatment, he travels along the left branch arriving at node A, a chance node. The outcome at this node is determined by the likelihood of a complication from renal arteriography and renal venous catheterization (e.g., atheromatous emboli, acute renal failure, infection). Assuming that no complications occur from these studies, he moves to node B, whose branches define the possible findings of the studies. (We will assume that the branches termed positive, equivocal and negative adequately represent the possible outcomes.) Here again, the choice of the branch is not his; the patient's currently unknown condition and the accuracy of the studies will determine the likelihood of a given outcome.

If we now follow the branch corresponding to a positive result of studies, we come to another chance node (node C), one branch from which defines an inoperable lesion and the other an operable one. The branch for an operable lesion leads, in turn, to a decision node (node 2) where the physician can employ either drugs or surgery. The choice of surgery leads to another chance node representing possible outcomes of such treatment (node D). (Again, a definition of the number of possible outcomes is a matter of judgment, a balancing between the objectives of manageability and realism.) If surgery results in cure and there are no surgical complications, nothing more needs to be done. However, if repair of the renal artery fails to control the blood pressure, the physician has the option of employing drugs (as shown at decision nodes 3 and 4), again with the possibility of several outcomes.

Most physicians will probably find the diagrammatic representation in Figure 1 quite in keeping with their thinking about medical problems. As it stands, however, the decision tree represents only options and possible consequences; it does not indicate the best decision. Such a decision is based on a variety of factors, which can be assigned to one of two categories: (1) the probabilities of the various outcomes, and (2) their values. Many of these factors are determined largely by the condition of the patient in question. Thus, although the options and outcomes remain the same for virtually all patients in whom, for example, arteriography is considered, the probabilities and values differ from one patient to the next, and the decision is influenced accordingly.

THE ANALYSIS OF DECISION TREES
Qualitative Analysis. Even the physician who organizes his decision making in a systematic and explicit fashion does not lay out in his mind a full tree of the type shown in Figure 1. Instead, he trims away the unimportant branches and consolidates others in order to reduce the problem to manageable proportions. Such a pruning process is ordinarily carried out in an instinctive and informal fashion, frequently without the expert explicitly describing (or even being able to describe) the process. However, fundamental principles can be made explicit and can provide the basis for logical pruning.

The main issues are the probabilities and values related to the branches in question. If both the probability of a given event and the risks as-
Figure 1. Decision tree describing possible actions by the physician, and their potential consequences, in a patient thought to have either essential hypertension or functionally significant renal artery stenosis. The circles define decision nodes (points at which the physician must choose an action), and the squares define chance nodes (points at which the outcome is not under the physician's control). For details see text.
Figure 2. A simplified version of the decision tree in Figure 1. Various portions of the larger tree have been deleted in keeping with the principles of qualitative analysis discussed in the text. Basically, the physician has used his judgment to eliminate or consolidate branches which represent either clearly unacceptable decisions or unlikely outcomes which can be ignored without significantly affecting the decision.

The following paragraphs describe the application of these principles to the decision tree presented in Figure 1. The results of this pruning process are shown in the simplified tree depicted in Figure 2.

The pruning process is best begun by examining the major chance and decision nodes on the most distal portion of a single trunk of the tree, trimming away or consolidating the appropriate twigs and then moving more proximally. For example, in Figure 1 the branches arising from node D describe the possible results of surgery. Although each of these branches is important to our thinking and must be preserved, the twigs distal to decision nodes 3 and 4 can be deleted because the decision to perform or not to perform surgery will not be based to any significant degree on the prospects of successful drug therapy should surgical treatment fail.

Let us now move back to the immediately proximal decision node (node 2) which defines the options open to the physician if the patient has a "positive-operable" lesion. Although both "drugs" and "surgery" are theoretic choices, it is evident that if we were willing to use drugs after having demonstrated the findings of functional renal artery stenosis (FRAS), we should not have undertaken the studies in the first place. Thus at node 2 we can logically prune drug therapy and all that follows it.

We now move back to chance nodes C and B. We decide that if functional renal artery stenosis is found (node C), but the arterial lesion proves to be inoperable or if the results of studies (node B) are either "negative" or "equivocal," we will not operate but will employ drug therapy. Thus we combine these three branches into a new branch "other" which is shown at node J in Figure 2 as leading to "drugs." (The possible outcomes of drug therapy are identical to those shown in detail for "drugs" in the right half of Figure 1.) We then move back to chance node A. If a serious complication of studies occurs, we assume that surgery would have to be either postponed or cancelled and that the patient would be treated with drugs, as shown at decision node 9 in Figure 2.

Now we turn to the right half of the tree in Figure 1 to the trunk labelled "drugs." As shown by the branches leading from chance node G, failure to control blood pressure may be associated with either the presence or absence of drug or of vascular complications. In each case, however, treatment failure confronts the physician with the decision as to whether to perform studies. Thus all outcomes from node G, including the more distal twigs, can be combined into two branches "studies" or "no studies." These two composite branches are shown in Figure 2 at node 10. Turning now to chance node F in Figure 1, we also see that if control of blood pressure is achieved but drug complications occur, we are, as in the case of failure of drug therapy, confronted with...
the choice of “studies” or “no studies.” This branch can therefore be pruned and merged with the two branches already originating from node 10 in Figure 2. Finally, “blood pressure control—no drug complications” in Figure 1 (node F, left branch) remains in Figure 2 as the left branch at node E. Note that the distal portions of this branch relating to “renal infarction” (node I) have been pruned from Figure 2 because the incidence of this complication is low and its consequences are ordinarily not very serious.

Obviously, this example simply illustrates an approach; individual physicians might well construct the tree and trim it in a different fashion. In these matters the knowledge and judgment of the clinician is critical. However, as mentioned earlier, the tree forces him to define his views in as explicit and rational a fashion as possible.

Use of the Decision Tree. Once the structure of a decision tree has been established, the next step is to introduce the specifics of the particular patient.

Case 1. As an example, consider a 24 year old man with a 3 month history of sustained hypertension. His family history is negative for hypertension. Examination shows a blood pressure of 240/140 mm Hg, hemorrhages and exudates in the eyegrounds, and the ventricular impulse displaced to the left. The electrocardiogram is consistent with left ventricular hypertrophy. The urinalysis is negative and the serum electrolyte values and vanillylmandelic acid (VMA) excretion are normal, but an intravenous pyelogram shows a 2 cm difference in kidney size (right greater than left) and delayed visualization on the left.

These clinical data can now be examined in terms of the pruned decision tree shown in Figure 2. It is immediately evident that the structure and detail of the tree are appropriate for this patient, but that the distal portion of the branches originating at node D can be further trimmed. On the basis of the information available the physician will strongly suspect the patient of having functional renal artery stenosis (FRAS). He recognizes that the patient may have essential hypertension (EH), but considers this diagnosis much less likely. Given this a priori assessment that the patient has functional renal artery stenosis, the physician knows first that if the lesion is operable (as is highly probable), surgery is very likely to control the hypertension [10,11] and second, that surgery in an otherwise healthy young man is unlikely to be associated with significant complications. On this basis he prunes three branches originating from node D in Figure 2, retaining only the branch “Blood pressure control, no surgical complications.”

Having particularized the tree for the patient under consideration, we now move back to node 1 of Figure 2 and ask ourselves what is the likelihood that studies will yield evidence of functional renal artery stenosis? Obviously, the greater the probability, the more willing we are to take whatever may be the risk of carrying out the necessary diagnostic studies. In this patient, all the evidence favors the diagnosis of functional renal artery stenosis; since the patient is young and otherwise healthy, there is little risk of the studies doing significant harm.

However, we must weigh the merits of the alternative course of omitting further studies and proceeding directly to drug therapy (node 1, right branch). We know that drug therapy is likely to produce good control of the blood pressure (node E, left branch), but we also know that treatment will almost certainly be necessary for many years and that such prolonged treatment carries with it significant risks. For example, a serious drug reaction may develop or the patient may abandon the tedious and expensive treatment program and hypertension may recur (node H, right branch) with associated vascular complications.

Before making a decision, the physician must also consider an important additional dimension of the problem, namely, the value of each form of treatment as it applies to the particular patient. For example, if the patient is little concerned about surgery and its attendant problems, and is, in addition, reluctant or unwilling to take medications indefinitely, the potential value of studies is great. On the other hand, if he is disinclined towards surgery with its risks, pain, cost and time lost from work, but is quite willing to take antihypertensive drugs under close medical supervision, the potential value of medical treatment is high. Thus a combination of values and probabilities must be utilized in order to optimize the decision-making process.

In light of these considerations what then is the best decision for our 24 year old patient with severe hypertension (Case 1)? If we assume that the patient is indifferent to the choice between drug therapy and surgery, then the decision seems quite straightforward. Given the history strongly suggestive of functional renal artery stenosis, the low risk of studies and of surgery, and the difficulties that can be anticipated with long-term therapy, most physicians would proceed with studies with a view to surgical intervention if the results were positive.

On the other hand, if the patient were 70 years old rather than 24, the decision might well change. In this instance (which we shall call Case 2), the risks of the arteriogram and surgery are
far greater, tending to make studies and possible surgical intervention less desirable. Furthermore, the attractiveness of medical treatment is greater than in Case 1 since the number of years over which drug therapy must be maintained (with its attendant risks, annoyance, cost, etc.) is presumably far less. Therefore, assuming indifference to choice of treatment on the part of the patient, the physician would probably not favor surgery in Case 2 even if functional renal artery stenosis were present, and he therefore will not recommend arteriography.* Thus the decision as to the first step is significantly influenced by consideration of all the eventual risks, consequences and values of outcomes.

In these two relatively uncomplicated cases, the physician’s medical knowledge combined with simple logic not only allowed him to prune the tree but also permitted him to reach a rational and clearcut clinical decision. Undoubtedly, each of the decisions came as no great surprise. On the other hand, the process by which the decision was obtained demands an orderly consideration of the diverse factors mentioned at the outset of the discussion, and therefore provides a framework and procedure that should be of even greater value in the many more difficult clinical problems that the physician often faces.

Quantitative Analysis. In some instances, a problem is so complex that an analysis carried out solely in the qualitative terms described will fail to weigh all the issues adequately. As mentioned earlier, there exist formal quantitative techniques for the analysis of complex trees, widely used in diverse areas of decision making but virtually ignored in medicine. Let us now consider the application of quantitative decision analysis to a patient with possible functional renal artery stenosis in whom the decision as to treatment is much more difficult than in the two cases cited. Before proceeding, however, it should be noted that the technique demands more of the physician than does his usual decision-making strategy. The argument for its use is that a serious and difficult problem deserves detailed consideration.

Case 3. Consider a 49 year old man with severe hypertension of 5 years’ duration. He had a myocardial infarction at age 45 and now has cardiac failure, well compensated on digitalis. Several members of his family are also known to have long-standing hypertension. On admission to the hospital his blood pressure was 250/150 mm Hg, and there was bilateral papilledema with hemorrhages and exudates in the eye-gounds. The patient was treated with numerous antihypertensive drugs. Blood pressure was ultimately partially controlled at a level of 190/120 mm Hg with methyldopa 3 g daily and chlorothiazide 1 g daily. Addition of other antihypertensive drugs to this regimen was either accompanied by complications (angina and cardiac failure) or resulted in no further lowering of blood pressure. The intravenous pyelogram showed kidneys of equal size. Serum creatinine concentration is 1.3 mg/100 ml. Serum electrolyte concentrations and VMA excretion were normal.

The physician believes it very likely that this patient has essential hypertension (EH), with perhaps only a 1 in 20 chance of functional renal artery stenosis. However, since the patient’s hypertension is only partially controlled, he now asks himself whether it is justifiable to undertake studies with the hope, albeit somewhat remote, that they will disclose a functional renal lesion that can be corrected by surgery.

He realizes that there are some complicating factors that argue against proceeding with studies and possible surgery. First, the presence of cardiovascular disease increases the risk of studies. Furthermore, even if the test result were positive, it could well be a false positive result and is, in fact, far more likely to be so than in a patient with a high a priori probability of functional renal artery stenosis. Finally, surgery in this patient would be appreciably more dangerous than in a younger patient without cardiovascular disease; thus the potential cost of a false positive result is high. In view of these problems it is not surprising that qualitative analysis fails to offer a clear decision.

Let us turn then to a consideration of the quantitative analysis of this complex decision problem. A decision tree for this patient, pruned in accordance with the principles of qualitative analysis, is shown in Figure 3. It is very similar to the tree in Figure 2, but only a single branch remains at node E because the patient is known to be only partially responsive to drug therapy. Note also that, as previously, it is assumed that a serious complication from studies precludes surgery (node A, right branch). It is also assumed that no additional information would be gained from repeated studies. As in Figure 2, the three results, “inoperable FRAS,” “equivocal” and “negative,” in Figure 1 have been lumped together at node J as “other” in a branch leading to drugs.

This simplified diagram represents the core of the particular patient’s problem and permits discussion of some functional principles of quantitative analysis. These same principles could, of course, be applied to a more detailed representa-
We will approach the problem in three steps: (1) We will focus on diagnosis and examine how our initial estimate of the probability that a patient has a given disease is influenced by the results of each test. (2) We will consider the quantification of probabilities. (3) We will consider the quantification of value judgments. We will then describe how these factors can be combined to give an expected value, a number that allows comparison in quantitative terms of the desirability of various courses of action. The outcomes are divided into two groups: those in which blood pressure control is achieved by surgery, and those in which drugs are employed either because surgery was not performed or because surgery failed to control the blood pressure. Blood pressure control is denoted by C, and subscripts are used to distinguish various states. Thus, C_surg. comp. and C_no surg. comp. denote the achievement of blood pressure control by surgery with and without serious surgical complications, respectively. Similar subscripts are used with PC to denote partial control of blood pressure.

Diagnosis. How do we revise a priori probabilities in the light of new information? To revise the likelihood of functional renal artery stenosis after studies, we must consider two factors: (1) What
was the a priori probability of the diagnosis based on previous clinical data? (2) How likely is the particular test to give a false positive or a false negative result? Obviously if on the basis of the initial data the patient is thought to have essential hypertension, and if the studies for functional renal artery stenosis are frequently falsely positive, the appraisal of a positive result would be different from that in a patient thought a priori to have functional renal artery stenosis.

To make this intuitive notion more useful it is necessary to be more precise. For example, functional renal artery stenosis may be incorrectly diagnosed if roentgenographic changes of renal artery stenosis are observed, and an error in the renin analyses demonstrates a seemingly marked difference in the renal venous renin concentrations when in fact no such difference exists. The frequency of such an error is uncertain; but presumably adequate analytic technic, it should not occur in more than a few per cent of cases. We must then ask whether such a chance of a false positive test really matters. Most physicians intuitively assume that, if the frequency of false positive studies is low, a positive test result virtually assures that a given patient has functional renal artery stenosis. Is this intuitive notion justified? To answer this question let us consider a group of 1,000 patients, 950 of whom have essential hypertension and 50 of whom have functional renal artery stenosis. From these values it follows that the chance that a patient with a positive test came from the functional renal artery stenosis group is

\[
\frac{19}{45 + 19} = 0.30.
\]

In other words, given that a large proportion of the patient population under consideration has essential hypertension (950 of 1,000), even a low incidence of false positive results (2 per cent) will mean that 30 per cent of the positive results will be false.

This diagrammatic approach can be formalized, and the need for the diagram can be obviated by the use of Bayes rule, a formal expression that deals with such probabilities [3,6]. Bayes rule can readily be used to revise one's a priori estimate of the likelihood of a diagnosis in light of new information. The general form of Bayes rule is as follows:

\[
P(D_1/S) = \frac{P(D_1)P(S/D_1)}{P(D_1)P(S/D_1) + P(D_2)P(S/D_2)}
\]

\[
P(D_1/S)\text{ is the probability of disease 1 given a certain diagnostic finding } S, P(D_1)\text{ is the a priori probability of disease 1, } P(S/D_1)\text{ is the probability of the diagnostic finding } S\text{ given that a patient has disease 1, etc.}
\]

Using similar notations we can use Bayes rule

* A striking illustration of the problem in relying on intuition to evaluate the importance of false positive and false negative test results is provided by an experiment carried out with groups of medical students, house officers and practicing physicians. A total of 290 subjects were asked to assume that a test for cancer was available that has the following characteristics: (1) The test is positive in 95 of 100 patients without cancer. They were also asked to assume that, on the average, 5 people in a population of 1,000 have previously undetected cancer.

The problem posed was as follows: if the test described is given to a randomly selected patient from this population and the test is positive, what is the probability that the patient actually has cancer? The great majority of the participants scored incorrectly, more than half of the physicians giving an answer a value of 50 per cent or greater. The correct answer is 9 per cent. It is notable that almost two thirds of the physicians involved in this study thought that the example was a reasonable analogy to medical problems that confront them daily.

* (1) \(P(\text{FRAS}/\text{POS.})\) is the probability of functional renal artery stenosis given a positive arteriogram and renal venous renin analysis. (2) \(P(\text{FRAS})\) is the a priori probability of functional renal artery stenosis. (3) \(P(\text{EH})\) is the a priori probability of essential hypertension. (4) \(P(\text{POS.}/\text{FRAS})\) is the probability of positive arteriogram and renal venous renin analyses given that the patient has functional renal artery stenosis (i.e., a true positive test). (5) \(P(\text{POS.}/\text{EH})\) is the probability of positive arteriogram and renal venous renin analyses given that the patient has essential hypertension (i.e., a false positive test).
Figure 5. Influence of the probability of false positive studies (renal arteriogram and renal venous renin assays) on the diagnostic certainty that a patient with positive studies has functional renal artery stenosis when \( P(\text{POS.}/\text{FRAS}) = 0.90 \). Each curve represents a different a priori probability that the patient has functional renal artery stenosis. Note that when this probability is great (e.g., \( P(\text{FRAS}) = 0.90 \)), then small probabilities of a false positive result in essential hypertension have little effect on the interpretation of the test. However, when the a priori probability of FRAS is low (e.g., \( P(\text{FRAS}) = 0.10 \)), the effect is pronounced.

As shown in Figure 5, we have used Bayes rule to calculate how \( P(\text{FRAS}/\text{POS.}) \) depends on the likelihood of a false positive test for four values of the a priori probability of functional renal artery stenosis ranging from 0.05 to 0.90 and for values of the incidence of false positive studies ranging from 0 to 0.05. Despite positive studies there is still a substantial likelihood that the patient does not have functional renal artery stenosis if the a priori probability of the patient having functional renal artery stenosis was low prior to studies. For example, in the circumstance in which the a priori probability of functional renal artery stenosis ranges between 5 and 10 per cent and the error in studies is 5 per cent, positive studies indicate only a 49 to 67 per cent probability that the patient indeed is suffering from functional renal artery stenosis.

Unfortunately, we frequently are not certain of how often a given test yields a false positive result. What value should we then use in our analysis? One approach is to assume various values in order to see how sensitive the decision is to this factor. If studies are the best decision for any "reasonable" value for false positive results, all is well. If, on the other hand, studies are best for one range of plausible values and drugs are best for another, then one range must be chosen. Still something has been gained, since deciding on a range is easier than selecting a precise value.

Quantification of Probabilities. How do we quantify the likelihoods of various risks and outcomes and then utilize such data? To illustrate the approach to quantification let us consider the risks associated with studies and with surgery. We have simplified this issue by considering only the serious complications of these procedures. For the patient in question (Case 3), we have assumed that the probabilities of such complications are 0.05 and 0.10 for studies and surgery, respectively. We also need to consider the probability of surgical control of the blood pressure, given a positive operable ("POS. OP.") result from the studies. Let us assume that the probability of an adequate surgical procedure is 0.99 given that the patient has operable functional renal artery stenosis. Since an adequate procedure will lead to control...
of blood pressure (C) only if the lesion is functional, then in notation similar to that on page 467,
\[ P(C/POS.OP.) = \frac{P(C/FRAS.OP)P(\text{FRAS.}/POS.OP.)}{P(C/FRAS.OP)P(\text{FRAS.}/POS.OP.} + \frac{P(C/\text{FRAS.}/POS.OP.})}{P(C/\text{FRAS.}/POS.OP.}) + \frac{P(C/\text{FRAS.}/POS.OP.})}{P(C/\text{FRAS.}/POS.OP.}) + \frac{P(C/\text{FRAS.}/POS.OP.})}{P(C/\text{FRAS.}/POS.OP.}) + \frac{P(C/\text{FRAS.}/POS.OP.})}{P(C/\text{FRAS.}/POS.OP.})\]

In the current case we have assumed that the probability that the lesion will be amenable to surgery is the same whether or not the lesion is functional, i.e., \( P(\text{OP.}/\text{FRAS.}) = P(\text{OP.}/\text{EH}) \).

Therefore:
\[ P(\text{FRAS.}/\text{POS.}) = P(\text{FRAS.}/\text{POS.}) \]

We have seen that for an a priori probability of functional renal artery stenosis of 0.05 and a false positive incidence of 0.02, the likelihood that a positive test means functional renal artery stenosis is 0.70 (Figure 4), and, therefore:
\[ P(\text{POS.}/\text{OP.}) = P(\text{POS.}/\text{OP.}) = (0.99)(0.70) = 0.69. \]

Furthermore, we know that blood pressure is partially controlled by drugs in this patient (Case 3), thus:
\[ P(\text{PC}/\text{POS.}) = 0.31. \]

The probabilities of the four branches from the chance node (node D) can be readily obtained. For example,
\[ P(\text{C}_{\text{surg. comp.}}) = P(\text{C}/\text{POS.})P(\text{surg. comp.}) = (0.69)(0.10) = 0.069. \]

Each branch is labelled with the appropriate probability.

Quantification of Value Judgments. How do we assign numerical values to the desirability or undesirability of various outcomes? An obvious approach to quantifying preferences for outcomes is for the clinician simply to list numbers which represent his evaluations, accounting for diverse considerations as well as possible. For example, for Case 3 the most preferred outcome of surgical intervention is “blood pressure control, no serious surgical complications,” and the least preferred is “partial blood pressure control with drugs, serious surgical complication.” All other outcomes fall between the two. Thus the clinician might assign a value of +5000 to \( C_{\text{no surg. comp.}} \), and −5000 to \( \text{PC}_{\text{surg. comp.}} \), with an appropriate intermediate value for each remaining outcome to reflect his preference for it relative to these two (Table I).

Experience has shown however that direct evaluation of preferences in this way often leads to inconsistencies; the decision maker often acts as though his preferences were different from those that he has already expressed quantitatively. In practice, the use of lotteries has clearly been demonstrated to provide greater consistency* [6].

Assuming the values as shown in Table I, we can now complete the quantitative analysis of the decision tree for this patient (Case 3, Figure 3). The basic idea is that the expected value (EV) of a chance node reflects both the probability and the value of each possible outcome. Thus, for example, the expected value of a chance node when there is a 0.5 chance of consequence 1 (of value 100) and a 0.5 chance of consequence 2 (of value 1000) is given by EV = 0.5 (100) + 0.5 (1000).

Consider now a third prospect, \( C_{\text{surg. comp.}} \). Suppose the clinician could guarantee this outcome or could take his chances with the lottery. Obviously he would prefer \( C_{\text{no surg. comp.}} \), (prize 1) to \( C_{\text{surg. comp.}} \), and therefore for \( p \) close to 1.0 he would choose the lottery. On the other hand, he would prefer \( C_{\text{surg. comp.}} \) to \( PC_{\text{surg. comp.}} \) (prize 2), and therefore for \( p \) close to 0, he would choose the guaranteed outcome. For some value of \( p \), say \( p' \), he will be indifferent between the guaranteed result and the lottery. In other words, the value of the two alternatives are equal. In terms of the formalism of utility theory, the expected value of control but with a surgical complication is given by:

\[ \text{EV}(C_{\text{surg. comp.}}) = p' \text{EV}(C_{\text{no surg. comp.}}) + (1 - p') \text{EV}(PC_{\text{surg. comp.}}) \]

This provides an independent check on the assumed value for \( C_{\text{surg. comp.}} \), relative to that for the two extreme outcomes.

It should be noted that we have assumed arbitrary values which summate the effects of all possible serious complications in order to simplify this example. In practice it might be necessary, of course, to assign probabilities and values to each of the possible serious complications of studies and surgery and to use these figures in making the quantitative analysis.
(1000) = 550. More generally, the expected value of a chance node with a "p1" chance of consequence 1, a "p2" chance at 2, etc., is

$$EV = p_1V_1 + p_2V_2 + \ldots + p_nV_n$$

when there are n possible consequences and \( V_n \) denotes the value of consequence n. Thus starting at the most distal chance node, chance node D, we face the possibility of four consequences, and

$$EV_D = p(C_{no surg. comp.})V(C_{no surg. comp.}) + p(PC_{surg. comp.})V(PC_{surg. comp.}) = (0.621)(5000) + (0.069)(-4000) + (0.279)(-2500) + (0.031)(-5000) = 1977.$$  

The more proximal node marked J can now be analyzed. It represents a 0.061 chance of reaching chance node D (of value 1977) and a 0.939 chance to reach \( PC_{no stud. comp.} \) (of value −1500). Therefore,

$$EV_J = (0.061)(1977) + (0.939)(-1500) = -1288.$$  

Moving back another stage to node A,

$$EV_A = (0.95)(-1288) + (0.05)(-4500) = -1488.$$  

This last value is the expected value of studies. Given that the expected value of drug therapy, \( PC_{drugs} \), is more favorable (i.e., −1000) in this case, drugs are preferred. Although this conclusion is based on the specific probabilities and values assumed for this case, the principles apply quite generally, i.e., the individual clinician with a different view of certain probabilities or values can utilize the same technic to integrate his own appraisal of the issues.

**COMMENTS**

To deal effectively with medical problems a clinician must not only be in command of a large body of facts, but must also be able to lay out the important alternatives for action and to weigh the risks and benefits which might follow. Decision analysis formalizes this process.* By asking the physician to lay out a decision tree it stimulates him to formulate the problem more completely than he otherwise might and forces recognition of the relationship between current decisions and possible future consequences. By pruning the nonessential elements, he can come to grips with the critical determinants of his action. Separating judgments of probabilities from those of values, permits rational analysis of a given course of action and indicates areas in which the physician's personal knowledge is insufficient and in which additional information must be sought in the medical literature. Furthermore, when the components of a decision are known, it is much easier to isolate the causes of differences of opinion among physicians and to determine whether more information could bring accord. A corollary is that the formal framework makes it easier to incorporate the opinion of another expert about his area of expertise. Also of great importance is the potential for accommodating the values of the patient and his family.

Observation of students and physicians suggests that only a limited number approach a problem in the way that has been described. Frequently instead, programmed responses are utilized ("the patient has severe hypertension, arteriography and renin studies are indicated; there is a coin lesion in the chest; excisional biopsy is indicated"). In many instances such responses represent the consensus as to how to approach most patients with a given problem, based on repeated analysis of the general situation by experts. To the extent that the patient is "typical," such an approach is sound. However, if the patient's problem deviates significantly from the norm, effective decision making demands a strategy for modifying the standard response. Decision analysis provides a means for dealing with these nonroutine situations.

Although the definition of a decision tree represents the important first step in formal decision analysis, the introduction of quantitative terms and the subsequent calculations of expected value represent the heart of the formal approach. Admittedly such an approach is more complex and time-consuming than the qualitative approach, but it offers important advantages in appropriate situations. One such situation is that associated with the decision to adopt a particular procedure as routine. Consider, for example, the routine use of gastroscopy to differentiate benign from malignant ulcer. The use of formal decision analysis for a number of typical patients should lead to a good basic understanding of the risks and benefits of this procedure and might well bring into focus the critical variables and permit the establishment of
guidelines for dealing with various classes of patients. Granted that the initial analysis for a number of typical ulcer patients will involve a considerable amount of thought, calculation and search of the relevant literature, such an investment in formal analysis should be well repaid by an improvement in the ability to deal quickly and rationally with subsequent analogous decisions.

Another situation in which decision analysis seems likely to be of value is when the potential risks and benefits are very large, and when the best course of action is not immediately clear. In such cases it is important to avoid simply following "intuition." As we have pointed out in the arteriography example earlier, disregarding the possibility of a false positive test on the grounds that "it only happens in a few per cent of the cases," although intuitively reasonable, is a potentially serious mistake. Most of us have poor intuitions about the appropriate weight to give evidence of this kind, but Bayes rule, by providing a formal way to assess such evidence, can be most valuable in dealing with this problem. Fortunately, in most situations the amount of computation required for application of Bayes rule is quite small.

Of course, objections can be raised to detailed quantification. One is that many of the data required by the analysis are "soft," but the assignment of specific values seems to imply precision. Certainly, this matter must be given continued consideration. On the other hand, decision analysis focuses attention on these data and our degree of confidence in them, and permits us to see how the choice of a particular probability or value influences our decision. This is a useful addition to our decision-making strategies.

One of the most persistent and important objections to the use of decision analysis in medicine is directed at the quantification of values. Many people are repelled by the notion that considerations of quality of life, pain, risk, etc. must be quantified. Our personal lives, however, are replete with judgments with respect to the balancing of apparently incommensurate prospects. If a person chooses to fly from Boston to New York rather than to take a train, he may be balancing increased convenience and reduced travel time against a slight increase in the probability of death through accident. Similarly, many people who believe in "statistics" continue to smoke, and others refuse to use automobile seat belts. Certainly there is no reason to believe that implicit judgments about preferences are to be favored over explicit ones in delivering medical care.

Our experience with teaching on the wards is that some physicians find that the qualitative approach to decision analysis assists them in formulating their thoughts and in analyzing a complex clinical problem. Indeed, after detailed definition of alternative courses of action, a student or house officer will often change his initial opinion and arrive at a decision consistent with that of expert consultants. On the other hand, a number of house officers and students argue that good decisions can almost always be made in an informal fashion, and that even if the technics of decision analysis are at times useful, they are too time-consuming and demanding, and other aspects of patient care command a higher priority. They also frequently argue that factual information and knowledge of pathophysiology are more important to the student, that the reward and evaluation system in medical schools is based largely on one's store of information rather than on one's decision-making capability. This latter argument is difficult to counter, since it does reflect much of the current approach to teaching and evaluation. It also brings into focus, however, the question of goals. Presumably our intent is to educate physicians to be good decision makers. What remains to be determined is what mix of teaching efforts directed towards factual knowledge on the one hand and the strategy of decision making on the other, is likely to produce the most effective clinicians.

The Computer and Analysis of Decision Trees. As a practical matter it seems unlikely that the clinicians on the wards will find the time necessary to carry out the quantitative formulation and analysis necessary for calculation of expected value as a basis for choice of a course of action. Perhaps a new generation of students versed in decision making will be willing to do so, but even that is uncertain. A much more promising approach to the use of decision analysis in clinical medicine is with the aid of the computer [12]. Decision trees can be pre-stored in their full detail, most of the relevant probabilities and value judgments can also be retained in computer memory (with the option of appropriate modification in light of the individual patient's particular medical and human characteristics) and computations can be carried out instantly with an immediate display of expected value. Moreover, judgmental pruning of the tree is not necessary since the computer can easily examine each branch and make an evaluation in an explicit fashion. A preliminary exploration of this computer-based approach is discussed elsewhere, as are the many problems that must be resolved if the technic is ultimately to have widespread value. For the moment, however, it is clear that the most practical use of decision analysis is likely to derive from the application of the qualitative approach to the complex problems which the physician frequently faces.
REFERENCES